SS 2009 2009/07/01

Exercises to the Lecture FSVT

Prof. Dr. Klaus Madlener sheet 10

Exercise 35: [Completion]

Let $E = \{x + 0 = x, x + s(y) = s(x + y), x + p(y) = p(x + y), x - 0 = x, x - s(y) = p(x - y), x - p(y) = s(x - y), s(p(x)) = x, p(s(x)) = x, ((x + y) - x) = y, (x + (y - x)) = y, ((x - y) + y) = x\}$

1. Complete E using any reduction ordering you like.

   Be verbose, write down for at least 5 most general unificators how you determined them when looking for critical pairs. Write down all critical pairs, you have looked at.

   Hint: Start with CPs of the last three equations.

2. Show, that completion will not succeed. Make a suggestion, what can be done on this problem.

Exercise 36: [Completion modulo $\sim$]

Let $>$ be a Knuth-Bendix-ordering with weight function φ defined by $\varphi(s) = 1$ for $s \in F \cup V$.

Let $E = \{f(x + y) \to f(x) * f(y), f(0) \to 1, x + 0 \to x, 0 + x \to x, x * 1 \to x, 1 * x \to x\}$ and $G = \{x + y = y + x, (x + y) + z = x + (y + z), x * y = y * x, (x * y) * z = x * (y * z)\}$.

Complete E modulo G with respect to $>$.

Exercise 37: [Implementation by equations]

Let $g : \mathbb{N}^{n+1} \to \mathbb{N}$, $h : \mathbb{N}^n \to \mathbb{N}$ be primitivly recursive functions and let $f : \mathbb{N}^n \to \mathbb{N}$ be defined by:

$$f(x_1, \ldots, x_n) = \mu_{z \leq h(x_1, \ldots, x_n)}[g(x_1, \ldots, x_n, z) = 0]$$

Give an equation set $G_{\hat{f}}$ and a function symbol $\hat{f}$, such that $\hat{f}$ implements the function f in $G_{\hat{f}}$.

Delivery: until 2009/07/08 FR:G08 Mo:G03,
by EMail to madlener@informatik.uni-kl.de