Exercises to the Lecture FSVT

Prof. Dr. Klaus Madlener sheet 7

Exercise 22:

Let $\leq\subseteq \text{Term}(F,V)\times\text{Term}(F,V)$ be defined as:

$$\begin{aligned} s \lesssim t &\text{ iff. exists a substitution } \sigma \text{ with } t\equiv\sigma(s)\\ s\approx t &\text{ iff. } s\lesssim t \text{ and } t\lesssim s\\ s<t &\text{ iff. } s\lesssim t \text{ and } s\not\approx t\end{aligned}$$

Show:

1. $<$ is strict part of a well-founded partial order. Which elements is this partial order defined on?
2. $s\approx t$ holds iff. a permutation ξ exists with $s\equiv\xi(t)$ (variable renaming).

Exercise 23:

This exercise is on an alternative specification of the integers INTEGER $=(sig,E)$ with

$$sig=(int,0,succ,pred,add),$$
$$E=\{succ(pred(x))=x, pred(succ(x))=x, add(0,y)=y, add(succ(x),y)=succ(add(x,y))\}$$

1. Show, that $(\mathbb{Z},0,+1,-1,+)$ is initial in Alg(INTEGER).
2. Structurize this specification using the specification INT. Show that INTEGER is an enrichment of INT.
3. Extend INTEGER by a function `absolute` with the properties of the absolute value function on $\mathbb{Z}$. Show that this is an enrichment of INT.

Exercise 24:

Let INT2 be the specification of integers from example 7.9 of the lecture. We combine INT2 with BOOL and $((\{\},\{<\}),E)$ to obtain a specification INT3, where

$E=\{<(0,\text{succ}(x))=\text{true}, <(\text{pred}(x),0)=\text{true}, <(0,\text{pred}(x))=\text{false}, <(\text{succ}(x),0)=\text{false}, <(\text{pred}(x),\text{pred}(y))=<(x,y), <(\text{succ}(x),\text{succ}(y))=<(x,y)\}$

1. Check, whether $T_{\text{INT3}}\,|_{\text{bool}}\cong$ Bool. Why would this be important? Hint: Look at $<(\text{succ}(\text{pred}(x)),\text{pred}(\text{succ}(y)))$.
2. Show that INT3 can not be fixed by additional equations.
3. Find further problems of INT3.

4. Make a suggestion for a specification INT4, such that $T_{\mathrm{INT4}}|_{\mathrm{int}} \cong \mathbb{Z}$, $T_{\mathrm{INT4}}|_{\mathrm{bool}} \cong$ Bool and $<$ is properly defined by its equations. Hint: Consider further function symbols.

Exercise 25:

Let specifications ELEMENT and NAT be given as:

`spec`	ELEMENT
`uses`	BOOL
`sorts`	E
`opns`	$\mathrm{eq} : \mathrm{E}, \mathrm{E} \to \mathrm{Bool}$
`vars`	$\mathrm{x}, \mathrm{y}, \mathrm{z} :\to \mathrm{E}$
`eqns`	$\mathrm{eq}(\mathrm{x}, \mathrm{x}) = \mathrm{true}$
	$\mathrm{eq}(\mathrm{x}, \mathrm{y}) = \mathrm{eq}(\mathrm{y}, \mathrm{x})$
	$\mathrm{eq}(\mathrm{x}, \mathrm{y}) = \mathrm{true}$ and $\mathrm{eq}(\mathrm{y}, \mathrm{z}) = \mathrm{true}$ implies $\mathrm{eq}(\mathrm{x}, \mathrm{z}) = \mathrm{true}$

`spec`	NAT
`uses`	BOOL
`sorts`	N
`opns`	$0 :\to \mathrm{N}$
	$\mathrm{s} : \mathrm{N} \to \mathrm{N}$
	$\mathrm{equal} : \mathrm{N}, \mathrm{N} \to \mathrm{Bool}$
`vars`	$\mathrm{n}, \mathrm{m} :\to \mathrm{N}$
`eqns`	$\mathrm{equal}(0, 0) = \mathrm{true}$
	$\mathrm{equal}(0, \mathrm{s}(\mathrm{n}) = \mathrm{false}$
	$\mathrm{equal}(\mathrm{s}(\mathrm{n}), 0) = \mathrm{false}$
	$\mathrm{equal}(\mathrm{s}(\mathrm{n}), \mathrm{s}(\mathrm{m})) = \mathrm{equal}(\mathrm{n}, \mathrm{m})$

Give a parametrized specification for sets over ELEMENT with the operations INSERT and REMOVE and prove:

1. The signature morphism σ : ELEMENT $\to$ NAT given by $\sigma(\mathrm{E}) = \mathrm{N}$ and $\sigma(\mathrm{eq} = \mathrm{equal})$ is no specification morphism.
2. $(T_{\mathrm{NAT}})|\sigma$ is a model of ELEMENT, i.e. it is a correct parameter assignment.
3. Does your specification satisfy $(T_{\mathrm{VALUE}})|_{\mathrm{NAT}} \cong T_{\mathrm{NAT}}$, i.e. is VALUE an extension of NAT? Is it an enrichment?

Delivery: until 2009-06-17, Fr: G07 Mo: G02,
by EMail to madlener@informatik.uni-kl.de