Exercises to the Lecture FSVT

Prof. Dr. Klaus Madlener sheet 8

Exercise 26:
Make yourself familiar with the chapter on abstract reduction systems. Use the literature. Make sure you know proofs of lemmata 8.3, 8.5, theorem 8.6, theorem 8.16, lemma 8.18.

Exercise 27:
Consider the mu-calculus with the following rules for arbitrary $X, Y \in \{m, i, u\}^*$:

$$\{\frac{Xi}{Xiu}, \frac{mY}{mYY}, \frac{XiiiY}{XuY}, \frac{XuuY}{XY}\}$$

1. Is the reduction system it is based on terminating?
2. Do $mi \to mu$, $mu \to mi$ resp. hold? Prove your claim.

Exercise 28: [Confluence and termination of rule sets over ground terms]
Let $R = \{(l_k, r_k)|k = 1, \ldots, n)\}$ be a finite rule set over ground terms. Prove:

1. If there is an infinite chain, then there is a rule $(l, r) \in R$ with an infinite chain from r.
2. If there is an infinite chain, then there is a j with $1 \leq j \leq n$and a ground term t, such that $l_j \stackrel{+}{\Rightarrow} t$ and l_j is a subterm of t.
3. Termination of R is decidable. (Termination is often denounced as 'Kettenbedingung' in german literature.)
4. Develop sufficient conditions for local confluence.

Exercise 29: [Knuth-Bendix-ordering]
Let $\varphi : F \cup V \to \mathbb{N}$ be a weight function with

$$\begin{aligned} &\varphi(x) = \alpha > 0 && \text{for all } x \in V && (1)\\ &\varphi(f) \geq \alpha && \text{if } f \text{ 0-ary} && (2)\\ &\varphi(f) > 0 && \text{if } f \text{ 1-ary} && (3)\\ &\varphi(f) \geq 0 && \text{else} && (4) \end{aligned}$$

Extend φ to $\varphi : \text{Term}(F, V) \to \mathbb{N}$ by

$$\varphi(f(t_1, \ldots, t_n)) = \varphi(f) + \sum_{i=1,\ldots,n} \varphi(t_i)$$

Define $s > t$ iff. $\varphi(s) > \varphi(t)$ and $|s|_x \geq |t|_x$ for all $x \in V$. Then $>$ is called a Knuth-Bendix-ordering. Prove for any Knuth-Bendix-ordering $>$:

1. $>$ is strict part of a wellfounded partial ordering
2. $>$ is compatible with substitution
3. $>$ is compatible with term replacement

Exercise 30:

Let

$$R_1 = \{F(0,1,x) \to F(x,x,x)\}$$
$$R_2 = \{G(x,y) \to x, G(x,y) \to y\}.$$

1. Prove: R_1 and R_2 are terminating.
2. Prove or disprove: The rule set $R_1 \cup R_2$ is terminating.

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