SS 2011 29. Juni 2011

Exercises for the Lecture Logics
Sheet 10

Prof. Dr. Klaus Madlener Delivery until 06. Juli 2011 10:00 Uhr

Exercise 1: [Interpretations, tutorial]
Let $I = (D, I_c, I_v)$ be the following interpretation:

$$\begin{aligned} D &= \mathbb{N} \\ I_c(1) &= 1_{\mathbb{N}} \\ I_c(2) &= 2_{\mathbb{N}} \\ I_c(p)(d) &= 1, \text{ iff } d \text{ is even} \\ I_c(q)(d_1, d_2) &= 1, \text{ iff } d_1 \mid d_2 \\ I_c(f)(d_1, d_2) &= d_1 \cdot d_2 \text{ (Multiplication in } \mathbb{N}) \\ I_v(x) &= I_v(y) = 3 \end{aligned}$$

with $d, d_1, d_2 \in D = \mathbb{N}$. Evaluate the following formulas under I

$$\begin{aligned} &A_1 \equiv \forall x\ [p(x) \leftrightarrow q(x, 2)] \\ &A_2 \equiv \forall x\ [(p(x) \wedge q(x, 2)) \rightarrow \forall y[p(f(x, y))]] \\ &A_3 \equiv \forall x\ [\forall y[q(y, x) \rightarrow (y = x \vee y = 1)] \rightarrow ((x \neq 2) \rightarrow \neg p(x))] \end{aligned}$$

Exercise 2: [equivalence, tutorial]
Prove or disprove:

1. $\forall x\ [p(x) \wedge q(x)] \models\!\!\!= \forall x\ [p(x)] \wedge \forall x\ [q(x)]$
2. $\exists x\ [p(x) \wedge q(x)] \models\!\!\!= \exists x\ [p(x)] \wedge \exists x\ [q(x)]$

Exercise 3: [important theorems, 6 P]
Prove:

1. $\{\forall x[3 \cdot x > 4], \exists x[p(x)], q(3 + 4)\} \models \forall x[42 > x] \rightarrow \exists x[p(x)]$
2. $\{p(a), p(x + 3) \rightarrow \exists y[y > p(x + 3)], p \vee q, p(x + 3)\} \models \exists y[y > p(x + 3)]$
3. $\{\forall x[3 \cdot x > 4], \exists x[p(x)], q(3 + 4)\} \models \forall y[\forall x[42 > x] \rightarrow \exists x[p(x)]]$
4. $\forall x[\neg(p(x) \rightarrow \exists y[q(f(a, b))])] \models\!\!\!= \neg(\forall y[p(y)] \rightarrow q(f(a, b)))$
5. $\Gamma, A \models \neg B$ iff $\Gamma, B \models \neg A$

For number 5.: Do not use the deduction theorem, no truth table do not write down "holds according to the lecture".

Exercise 4: [PKNF, PDNF, 2+2P]
Transform A_1 into PKNF and A_2 into PDNF:

$A_1 \equiv \forall x[p(x) \to q(f(b)) \vee \forall y \exists z[r(f(x), g(z)) \to (p(z) \wedge r(z, x))]]$
$A_2 \equiv \forall x \forall y[(p(x) \to q(z)) \to \exists z[r(x, z) \leftrightarrow q(x)]]$

Exercise 5: [equivalence and conclusion, 6P]
Prove or disprove:

1. $\forall x\ A \models\!\!\!\Rightarrow \neg\exists x\ \neg A$
2. $\forall x\ [p(a) \to q(x)] \models\!\!\!\Rightarrow p(a) \to \forall x\ q(x)$
3. $\exists x\ [p(x) \wedge q(x)] \models\!\!\!\Rightarrow \exists x\ [p(x)] \wedge \exists x\ [q(x)]$
4. $\models \exists x\ [p(x) \wedge q(x)]$ iff. $\models \exists x\ [p(x)] \wedge \exists x\ [p(x)]$
5. $\models \forall x\ [(p(a) \wedge \neg q(b, c)) \to (q(b, c) \to p(a))]$
6. $\models \forall P \exists Q\ [P \leftrightarrow \neg Q]$

Delivery: until 06. Juli 2011 10:00 Uhr into the box next to room 34-401.4