SS 2011 13. Juli 2011

Exercises for the Lecture Logics
Sheet 12

Prof. Dr. Klaus Madlener Delivery until 20. Juli 2011 10:00 Uhr

Exercise 1: [Tableauxfolgerung, Übung]
Es sei

$$\Sigma = \{\forall x \forall y \forall z\ x \cdot (y \cdot z) = (x \cdot y) \cdot z, \quad \forall x\ 1 \cdot x = x, \quad \forall x\ x \cdot x = 1\}.$$

Zeigen Sie $\Sigma \models \forall x\ x \cdot 1 = x$ mit der Tableaux-Methode.

Exercise 2: [Formalisation with Tableaux, Übung]
Consider the following propositions:

- Every policeman is determined.
- If you are determined and intelligent, then you will do your job well.
- George is an intelligent policeman.
- Therefore George will do his job well.

1. Formalise the Propositions in PL1.
2. Use a tableau to show that the last proposition is a conclusion of the first three.
3. Use a tableau to construct a model for the first three propositions.

Exercise 3: [Tableaux and models, tutorial]
Use a tableaux to construct a satisfying interpretation for

1. $\{\exists x \exists y \exists z\ (\neg x = y \wedge \neg x = z),\ \forall x\ x = x\}$

Exercise 4: [Tableaux, 6P]
Prove using the tableaux-Method:

1. $\vdash_\tau \forall x[A(x)] \leftrightarrow \forall y[A(y)]$
2. $\forall x[A(x) \rightarrow B(x)] \vdash_\tau \exists x[A(x) \rightarrow \exists x[B(x)]]$
3. $\vdash_\tau \forall x \forall y[\neg p(x) \rightarrow (((x = y) \rightarrow (p(x) \rightarrow p(y))) \rightarrow (((x = y) \rightarrow p(x)) \rightarrow ((x = y) \rightarrow p(y))))]$

Exercise 5: [Tableaux and models, 4P]
Use a tableaux to construct a satisfying interpretation for:

1. $\exists x \exists y[x \neq y \wedge \forall z[z = x \vee z = y]]$
2. $\exists x \forall y[p(x) \rightarrow p(y)]$

Exercise 6: [Soundness of the Tableaux-rules, 8P]

Let γ- and δ-formulas be defined as in the lecture and let I be an interpretation, such that for every $d \in D$ there is a term t with $I(t) = d$. Prove:

- If $\gamma[t]$ is unsatisfiable, then γ is unsatisfiable as well.
- If $I \models \{\gamma[t] \mid t \text{ ist Term}\}$, then $I \models \gamma$.
- If $\delta[y]$ is unsatisfiable, then δ is unsatisfiable as well.
- $\delta[y] \models \delta$.

Delivery: until 20. Juli 2011 10:00 Uhr into the box next to room 34-401.4