SS 2011 20. April 2011

Exercises for the Lecture Logics
Sheet 1

Prof. Dr. Klaus Madlener Delivery until 27. April 2011 10:00h

Exercise 1: [structural induction, tutorial]
Prove by strucural induction:

1. Every propositional formula contains at least one symbol that is different from $\neg$.
2. In every propositional formula the number of pairs of brackets equals the number of operators.
3. Let n be the number of occurrences of variables in $A \in F$. The number of operators in A is at least $n - 1$.

Exercise 2: [Valuation of formulas, tutorial]
Let $p, q \in V$ be propositional variables. Prove by considering all valuations:

1. $A_1 \equiv (p \to q) \vee (q \to p)$ is a tautology, i.e. formula that is satisfied by all assignments.
2. $A_2 \equiv (p \to q) \wedge (q \to p)$ is satisfiable, i.e. there is a satisfying assignment.

Exercise 3: [Relations between formal and informal logics, 8P]
Try to express the following propositions in terms of logical formulas.

1. "Students eat in the mensa."
2. "In the bus the old ones sit, the young ones stand."
3. "People who counterfeit or alter bank notes or procure and place into circulation counterfeited or altered bank notes will be sent to prison for at least two years."
4. "William Shakespeare has written 'Moby Dick' and Paris is the spanish capital and cats chase mice."
5. "On Fridays we eat saddle steak."
6. "This sentence consists of six words."
7. "This sentence doesn't consist of six words."
8. "If there is a day when all the meals in the mensa are good, then there is also a day when none of the meals are good."

Which of these propositions are "true", which ones are "false". Briefly discuss the occurring problems.

Example: To formalise the sentence "If I am not at home you can call me on my cell phone.", we could define two atomic formulas

- $A \equiv$ „I am at home.“ and

- $B \equiv$ „You can call me on my cell phone."

The proposition is then represented by the formula $(\neg A) \rightarrow B$. The proposition is "false". (Who is meant by "I" and "you"?)

Exercise 4: [Relations between formal and informal logics, 6P]

A 100-year old was asked: "What is the secret of your long life?". He answered: "I strictly stick to the following diet: if I don't drink beer with a meal, then I always have fish. Whenever I have fish and beer together, then I don't have ice cream. If I eat ice cream or avoid beer, then I never eat fish."

Formalise the diet using propositional formulas and try to find a simpler description.

Exercise 5: [structural induction, 8P]

1. Prove that every propositional formula is finite.
2. Inductively define the set G of all propositional formulas that contain only the operators $\neg, \wedge, \vee$ and in which every variable is negative (but only the variables, i.e. no negated composite formulas). Prove by structural induction that every formula $A \in G$ contains exactly as many unitary operators as there are variables.
3. Consider the following inductively defined sets of formulas:

   F':
   1. $p_i \in F'$ for all $i \in \mathbb{N}$
   2. If $A, B \in F'$, then $A \wedge B$ and $A \vee B \in F'$.
   3. F' is the smallest set satisfying 1. and 2..

   F'': zusammengesetzt
   1. $F' \subset F''$
   2. If $A, B \in F''$, then $\neg A$, $A \rightarrow B$ and $A \leftrightarrow B \in F''$.
   3. F'' is the smallest set satisfying 1. and 2..

   Describe the types of formulas contained in these sets in your own words and give examples of formulas that are contained resp. not contained in the sets.

Exercise 6: [Valuation of formulas, 6P]

Which of the following propositional formulas are tautologies, satisfiable or unsatisfiable?

$A_3 \equiv \neg(((p \to q) \wedge p) \to q)$

$A_4 \equiv \neg((\neg p \vee q) \wedge p) \vee q$

$A_5 \equiv p \to \neg p$

$A_6 \equiv (\neg p \vee q) \wedge (\neg q \vee r) \wedge (\neg r \vee p)$

$A_7 \equiv (\neg p \wedge q) \vee (\neg q \wedge r) \vee (\neg r \wedge p)$

$A_8 \equiv p \to (q \to p)$

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