SS 2011 | 11. Mai 2011

Exercises for the Lecture Logics
Sheet 4

Prof. Dr. Klaus Madlener | Delivery until 18. Mai 2011 10:00 Uhr

Exercise 1: [deductive systems, tutorial]
Let the deductive system $\hat{\mathcal{F}}$ be defined by changing the first axiom pattern of $\mathcal{F}_0$ to

$$A \to (A \to B).$$

1. Is $\hat{\mathcal{F}}$ complete?
2. Ist $\hat{\mathcal{F}}$ sound?

Exercise 2: [Proofs in deductive systems, tutorial]
Prove:

1. $(\neg(p \to q)) \vdash_G (q \to p)$
2. $(\neg(p \to q)) \vdash_H (q \to p)$
3. $\vdash_G (p \land q) \to (p \lor r)$
4. $\vdash_H (p \land q) \to (p \lor r)$

Exercise 3: [Soundness of Gentzen's sequent calculus, 4P]
Prove that the sequent calculus is sound, i.e.

$$\text{if } \Gamma \vdash_G \Delta \text{ then } \Gamma \models \Delta.$$

Exercise 4: [Completeness of the Hilbert calculus, 8P]
Prove that the Hilbert calculus is complete.

If you use a rule that is not explicitly mentioned on slides 70f, then define it yourself and argue about its soundness.

Exercise 5: [Proofs in deductive systems, 8P]
Prove:

1. $\vdash_G (A \to (B \to C)) \to ((A \to B) \to (A \to C))$
2. $A \to B, C \to D, A \lor C \vdash_G B \lor D$
3. $\vdash_G \neg(A \lor B) \lor B \lor A$
4. $A \land B, B \land C \vdash_G A \land C$

Delivery: until 18. Mai 2011 10:00 Uhr into the box next to room 34-401.4