SS 2011 18. Mai 2011

Exercises for the Lecture Logics
Sheet 5

Prof. Dr. Klaus Madlener Delivery until 25. Mai 2011 10:00 Uhr

Exercise 1: [Tableaux, tutorial]
On sheet 1 a diet plan was described by the following formula:

$$A \equiv (\neg B \to F) \wedge (((B \wedge F) \to \neg E) \wedge ((E \vee \neg B) \to \neg F))$$

Construct a complete tableau for A. What properties of A can be derived using the tableau? Find a disjunctive normal form for A using the tableau.

Exercise 2: [Tableaux-consequence, tutorial]
Prove:

1. $(A \wedge \neg B) \vdash_{\tau} \neg((\neg A) \wedge (\neg B))$
2. $(A \wedge (A \to B)) \vdash_{\tau} B$
3. $A \to (B \to C) \vdash_{\tau} (A \to B) \to (A \to C)$

Exercise 3: [compactness theorem, 6P]

1. Prove that for a set of formulas Σ and a formula A there is a closed tableau for $\Sigma \cup \{\neg A\}$ iff there is a closed tableau for $\Gamma \cup \{\neg A\}$ for every finite subset $\Gamma \subset \Sigma$.

   Hint: You can use König's Lemma, which states that a tree is finite iff every branch is finite.
2. Prove that the compactness theorem holds for tableaux, i.e.

$$\Sigma \vdash_{\tau} A \text{ iff there is a finite subset } \Sigma_0 \subseteq \Sigma \text{ with } \Sigma_0 \vdash_{\tau} A.$$

Exercise 4: [Tableaux-consequence, 5P]
Prove:

1. $\{p, p \vee q, p \to s, r \to q\} \vdash_{\tau} q \to p$
2. $\{p, p \vee q, p \to s, r \to q\} \vdash_{\tau} s$
3. $\vdash_{\tau} (\neg(p \to q) \to (q \to p))$
4. $F \vdash_{\tau} q \to p \wedge (\neg(s \wedge \neg(s \vee ((q \wedge r) \to p))))$
5. $\neg((A \to (A \vee C)) \wedge D) \vdash_{\tau} (C \to B) \vee \neg D$

Exercise 5: [DNF from tableaux, 2P]
Using the tableaux-method, find a DNF for the following formulas:

1. $(p \to \neg(\neg q \to r)) \to (q \lor r)$
2. $(q \to p) \to \neg(r \to q)$

Exercise 6: [Tableaux with equivalence, 4P]

α- and β-formulas have so far only been defined for $\{\neg, \land, \lor, \to\}$, but $\leftrightarrow$ has been omitted. Is $A \equiv B \leftrightarrow C$ an α- or a β-formula and which components does it consist of?

Exercise 7: [limitations of the tableau method, 5P]

Can the following statements be proved using the tableaux method? Give short explanations.

1. $\{p, q, r, s\} \models t$
2. $\{p, q, r, s\} \not\models \neg(q \to s)$
3. $F \models \neg(p \to (q \leftrightarrow r) \land \neg r) \to (s \lor \neg p)$
4. $\Sigma := \{p_i \land \neg p_{i+1} | i \in \mathbb{N}\}$ is unsatisfiable.
5. $\Sigma := \{p_i \land \neg p_{i+1} | i \in 2\mathbb{N}\}$ is satisfiable.

Keep your arguments as general as possible. I.e. if you can explain why there is a tableaux-proof without actually writing it down, then do not write it down.

Exercise 8: [Tableaux-consequence, tutorial]

Prove without using the soundness and completeness of the tableaux-method:

If $\Sigma \vdash_{\mathcal{T}} p \land q$ then $\Sigma \vdash_{\mathcal{T}} p \lor q$ holds as well.

Delivery: until 25. Mai 2011 10:00 Uhr into the box next to room 34-401.4