SS 2011 25. Mai 2011

Exercises for the Lecture Logics
Sheet 6

Prof. Dr. Klaus Madlener Delivery until 1. Juni 2011 10:00 Uhr

Exercise 1: [Negation normal form, tutorial]
Convert the following formulas into Negation normal form:

1. $A_1 \equiv p \wedge ((\neg q \to r) \leftrightarrow (\neg r \vee p))$
2. $A_2 \equiv (p \to (q \to r)) \to ((p \to q) \to (p \to r))$
3. $A_3 \equiv (\neg p_1 \wedge \neg p_2) \vee (p_1 \wedge \neg p_2) \vee (p_2 \wedge \neg p_3) \vee (p_1 \to p_3)$
4. $A_4 \equiv \neg(p_1 \to (\neg p_2 \wedge \neg p_3 \wedge \neg p_5)) \wedge (p_2 \to (p_4 \to p_3)) \wedge \neg((p_2 \wedge p_4) \vee (\neg p_2 \wedge p_5))$

Exercise 2: [Davis-Putnam, tutorial]
Use the Davis-Putnam-Algorithm to check the formulas from exercise 1 for satisfiability.

Exercise 3: [dual formulas, 5P]
Let $A \in F(\{\neg, \vee, \wedge\})$ and $d(A)$ be the dual formula of A. Let further be φ a valuation and $\varphi'(p) := 1 - \varphi(p)$ for all $p \in V$. Prove that $\varphi'(d(A)) = 1 - \varphi(A)$.

Exercise 4: [Davis-Putnam, 4P]
Prove using the Davis-Putnam-Method:

1. $p \wedge q, q \to r \models r$
2. $p \to r, q \to s, p \vee q \models r \vee s$
3. $\neg q, p \to q \models \neg p$
4. $\models \neg(p \to q) \to (q \to p)$

Exercise 5: [Pure-Literal-Rule, 8P]

1. Let A be a formula in negation normal form, where p occurs only positively. Prove by structural induction:
$$A[p/0] \models A[p/1].$$
2. Conclude that A is equisatisfiable with $A[p/1]$.
3. Find a formula $A \in F(\{\neg, \wedge, \vee\})$ to which the rule can be applied and for which $A[p/1]$ resp. $A[p/0]$ are not equisatisfiable with A.

Delivery: until 1. Juni 2011 10:00 Uhr into the box next to room 34-401.4