

Exercises for the Lecture Logics
Sheet 7

Prof. Dr. Klaus Madlener

Delivery until 08. Juni 2011 10:00 Uhr

Exercise 1: [Resolution, tutorial]

Prove by resolution:

1. $A \rightarrow (B \rightarrow A)$ is a tautology.
2. $\{p \vee q, q \vee r\} \models p \vee r$.
3. $X \equiv (\neg B \rightarrow F) \wedge (((B \wedge F) \rightarrow \neg E) \wedge ((E \vee \neg B) \rightarrow \neg F))$ is satisfiable.

Exercise 2: [PL-Warmup, tutorial]

1. Evaluate the formulas $U(x)$ and $G(x)$ from slide 140 for $x = 2, 3$ and 4 (in \mathbb{Z}).
2. Find a formula $T(x, y)$ that expresses the "divides"-Operator in \mathbb{Z} .

Exercise 3: [Resolution, 10P]

Prove by resolution:

1. $p \models p \vee q$.
2. $\{p \vee q, \neg q \vee r\} \models p \vee r$.
3. $((p \rightarrow q) \rightarrow p) \rightarrow p$ is a tautology.
4. $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow \neg(\neg r \wedge p)$ is a tautology Tautology.
5. $\neg((\neg p \rightarrow (q \vee r)) \wedge \neg((p \wedge \neg q) \vee (r \wedge \neg s \wedge t)) \wedge (q \rightarrow s) \wedge \neg(s \vee \neg t))$ is a tautology.

Exercise 4: [Satisfying valuations and resolution, 4P]

Prove by resolution:

1. $(\neg p \vee q) \wedge (\neg q \vee r) \wedge (\neg r \vee s) \wedge (\neg s \vee p)$ is satisfiable.
2. $(p \rightarrow q) \wedge (\neg q \vee r) \not\models \neg r$

Exercise 5: [Properties of resolution, 7P]

1. Prove the soundness of the resolution calculus.
2. Prove that clauses that are subsumed by others are not necessary for any resolution step.

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