Exercises for the Lecture Logics
Sheet 8

Prof. Dr. Klaus Madlener   Delivery until 23. Juni 2011 10:00 Uhr

Exercise 1: [Terms and formulas, tutorial]
  Consider the following expressions:

  a) $3 + 4$

  b) $x \cdot 7$

  c) $3 + x \geq 28$

  d) if $3 + x \leq 28$ then 5 else 7

  e) if $3 + x \geq 28$ then $5 = 8$ else 7

  f) if $3 + x \geq 28$ then (if $x > b$)then $5 > 3$ else $3 > 5$) else $p(x \cdot (3 + 4))$

  g) p_1

  h) $((\forall x)((\exists y)(x \geq y)))$

  i) $\forall F[F(x) = 0 \rightarrow 3 + 4 \geq 7]$

The following tasks are meant to get you accustomed with the notions of Terms, formulas, interpretations etc.

  1. Which of the aforementioned expressions are terms and which are formulas?

  2. Which (sub-)formulas are atomic?

  3. Identify the function- and predicate variables and note down their arity.

  4. Which variables are bound, which ones are free?

  5. For every formula, find an interpretation that satisfies the formula.

  6. For every formula, find an interpretation that does not satisfy the formula.

Exercise 2: [Formalisation in PL, 8P]
  Consider the followinf expressions:

  a) „There is at least one dragon."

  b) „Heroes kill dragons."

  c) „Siegfried is a hero and he takes a bath in a dragon's blood."

  d) „Bathing in a dragon's blood makes you invulnerable."

  e) „Siegfried marries Kriemhild."

  f) „There are at least two different men whom Kriemhild marries."

  g) „Hagen is neither a hero nor a dragon and he kills Siegfried."

  h) „Etzel is Kriemhilds second husband."

  i) „Kriemhild kills her brother."

j) „In the end everyone is dead."

Formalise this short version of the "Nibelungen Saga" using first-order-formulas:

1. Find suitable function and predicate symbols.
2. Find formulas that express the above propositions.
3. Shortly discuss the arising problems.

Exercise 3: [Interpretations, 6P]

Define an interpretation I that satisfies the following Formulas:

1. $\exists x \forall y \ y + x = 0$,
2. $\forall x \forall y \forall z \ x + (y + z) = (x + y) + z$,
3. $\forall x \forall y \ (x + y = y + x) \rightarrow x = y$ und
4. $\exists x \ x + 5 < x$.

0 and 5 are individual constants, $+$ is a binary function symbol and $<$ is a binary predicate symbol.

Exercise 4: [Universality, 5P]

Which of the following formulas are universally valid?

1. $(x = y \wedge y = z) \rightarrow x = z$,
2. $(x < y \wedge y < z) \rightarrow x < z$,
3. $\forall Q \ Q(x) \rightarrow \neg x = x$,
4. $(p(x) \rightarrow q(x)) \rightarrow \forall x \ (p(x) \rightarrow q(x))$ und
5. $\exists x \ (p(x) \rightarrow \forall y \ p(y))$.

Exercise 5: [structural induction in PL, 6P]

Let $I_1 = (D_1, I_{C_1}, I_{V_1})$ and $I_2 = (D_2, I_{C_2}, I_{V_2})$ be interpretations with $D_1 = D_2$. Show that the following holds for every formula A:

If I_1 and I_2 coincide on all constants and free variables of A, then $I_1(A) = I_2(A)$ holds.

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