Logical calculi

- Syntactical manipulations based on transformation rules
- Starts off from a set of formulas and a set of axioms
- New formulas can be derived using deduction rules from the given set of formulas Γ
- Φ is provable if it can be obtained with a finite number of derivation steps from the formulas in Γ
  - $\Gamma \vdash \Phi$: Φ is provable from Γ
  - $\vdash \Phi$: Φ is provable from a set of axioms

Properties of proof systems

- Soundness
  - A proof system is sound if, for any formula Φ
    - $\Gamma \vdash \Phi$ implies $\Gamma \models \Phi$
  - A formula that is provable is true

- Completeness
  - A proof system is complete if, for any formula Φ
    - $\Gamma \models \Phi$ implies $\Gamma \vdash \Phi$
  - Every true formula is provable

Proof system styles

- Hilbert style
  - Easy to understand (only one rule)
  - Hard to use

- Natural deduction
  - Easy to use
  - Hard to understand (many rules)
  - Used in Isabelle

Hilbert calculus

- Large number of axioms and small set of rules of inference
- The most commonly studied Hilbert systems have just one rule of inference – modus ponens
- For the exercise: 3 axioms and modus ponens

Natural deduction

- Kind of proof calculus in which logical reasoning is expressed by inference rules closely related to the "natural way” of reasoning
- Based on introducing hypothesis as natural steps in proofs
- Contrasts with the axiomatic systems (like Hilbert's) which instead use axioms
- Rich set of rules
- One axiom only