Exercises to the Lecture FSVT

Prof. Dr. Klaus Madlener   sheet 7

Exercise 20:

1. Give models for the specifications NAT and LIST(NAT) from the lecture, where the sets of support consist of ground terms.

2. Give models for the specifications NAT and LIST(NAT), with + not commutative, and app not associative.

   Are your sig-algebras term-generated?

Exercise 21:

Let the specification LIST(NAT) $= (\mathrm{sig}, E)$ be the specification of lists from the lecture.

1. Show, that for every ground term, there is a E-equal ground term, not containing app.

2. Show: $\mathrm{app}(q_1, \mathrm{app}(q_2, q_3)) = \mathrm{app}(\mathrm{app}(q_1, q_2), q_3) \in \mathrm{ITH}(E)$

Exercise 22:

Prove:

1. Let $t, t', t'' \in \mathrm{Term}(F, V), u \in O(t), v \in O(t')$. Then holds:

$$t[u \leftarrow t']/uv \equiv t'/v \qquad\qquad \text{(embedding)}$$
$$t[u \leftarrow t'][uv \leftarrow t''] \equiv t[u \leftarrow t'[v \leftarrow t'']] \qquad\qquad \text{(associativity)}$$

   or in alternative syntax:

$$t[t']_u \mid_{uv} \equiv t' \mid_v \qquad\qquad \text{(embedding)}$$
$$t[t']_u[t'']_{uv} \equiv t[t'[t'']_v]_u \qquad\qquad \text{(associativity)}$$

2. Let $t, t', t'' \in \mathrm{Term}(F, V), u, v \in O(t), u \mid v$ (u, v are disjunct positions, i.e. neither u is prefix of v nor v prefix of u). Then holds:

$$t[u \leftarrow t']/v \equiv t/v \qquad\qquad \text{(persistence)}$$
$$t[u \leftarrow t'][v \leftarrow t''] \equiv t[v \leftarrow t''][u \leftarrow t'] \qquad\qquad \text{(commutativity)}$$

3. Let $t, t', t'' \in \mathrm{Term}(F, V), u, v, w \in O(t), u = vw$. Then holds:

$$t[u \leftarrow t']/v \equiv (t/v)[w \leftarrow t'] \qquad\qquad \text{distributivity}$$
$$t[u \leftarrow t'][v \leftarrow t''] \equiv t[v \leftarrow t''] \qquad\qquad \text{(dominance)}$$

Delivery: until 2007-12-06, by EMail to Bernd Strieder