

Exercises to the Lecture FSVT

Prof. Dr. Klaus Madlener

sheet 8

Exercise 23:Let $\leq \subseteq \text{Term}(F, V) \times \text{Term}(F, V)$ be defined as: $s \lesssim t$ iff. exists a substitution σ with $t \equiv \sigma(s)$ $s \approx t$ iff. $s \lesssim t$ and $t \lesssim s$ $s < t$ iff. $s \lesssim t$ and $s \not\approx t$

Show:

1. $<$ is strict part of a well-founded partial order. Which elements is this partial order defined on?
2. $s \approx t$ holds iff. a permutation ξ exists with $s \equiv \xi(t)$ (variable renaming).

Exercise 24:This exercise is on an alternative specification of the integers $\text{INTEGER} = (sig, E)$ with

$$sig = (int, 0, succ, pred, add),$$

$$E = \{succ(pred(x)) = x, pred(succ(x)) = x, add(0, y) = y, add(succ(x), y) = succ(add(x, y))\}$$

1. Show, that $(\mathbb{Z}, 0, +1, -1, +)$ is initial in $\text{Alg}(\text{INTEGER})$.
2. Structurize this specification using the specification INT . Show that INTEGER is an enrichment of INT .
3. Extend INTEGER by a function `absolute` with the properties of the absolute value function on \mathbb{Z} . Show that this is an enrichment of INT .

Exercise 25:Let $sig_1 = (\{\text{NAT}, \text{EVEN}\}, \{0, 1, S, f\}, \{0 : \rightarrow \text{NAT}, 1 : \rightarrow \text{EVEN}, S : \text{NAT} \rightarrow \text{NAT}, f : \text{NAT} \rightarrow \text{EVEN}\})$. Further, let the sig_1 -Algebra \mathfrak{A}_1 be defined by:

$$A_{1, \text{NAT}} = \mathbb{N}, A_{1, \text{EVEN}} = 2\mathbb{N} \cup \{1\}, 0_{\mathfrak{A}_1} = 0, 1_{\mathfrak{A}_1} = 1, S_{\mathfrak{A}_1}(x) = x + 1, f_{\mathfrak{A}_1}(x) = \begin{cases} x, & \text{if } x \text{ even} \\ 1, & \text{else} \end{cases}$$

Prove:

1. There is no specification $spec_1 = (sig_1, E_1)$ with finite E_1 , such that $T_{spec_1} \cong \mathfrak{A}_1$.
2. There is a specification $spec_2 = (sig_2, E_2)$ with $sig_1 \subseteq sig_2$, E_2 finite, such that $T_{spec_2}|_{sig_1} \cong \mathfrak{A}_1$.

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