WS 2007/2008 2007-12-14

Exercises to the Lecture FSVT

Prof. Dr. Klaus Madlener sheet 9

Exercise 26:

Let INT2 be the specification of integers from example 7.9 of the lecture. We combine INT2 with BOOL and $((\{\}, \{<\}), E)$ to obtain a specification INT3, where

$E = \{< (0, \text{succ}(x)) = \text{true}, < (\text{pred}(x), 0) = \text{true}, < (0, \text{pred}(x)) = \text{false}, < (\text{succ}(x), 0) = \text{false}, < (\text{pred}(x), \text{pred}(y)) =< (x, y), < (\text{succ}(x), \text{succ}(y)) =< (x, y)\}$

1. Check, whether $T_{\text{INT3}} \mid_{\text{bool}} \cong$ Bool. Why would this be important? Hint: Look at $< (\text{succ}(\text{pred}(x)), \text{pred}(\text{succ}(y)))$.
2. Show that INT3 can not be fixed by additional equations.
3. Find further problems of INT3.
4. Make a suggestion for a specification INT4, such that $T_{\text{INT4}}|_{\text{int}} \cong \mathbb{Z}$, $T_{\text{INT4}}|_{\text{bool}} \cong$ Bool and $<$ is properly defined by its equations. Hint: Consider further function symbols.

Exercise 27:

Let spezifications ELEMENT and NAT be given as:

```
spec    ELEMENT
uses    BOOL
sorts   E
opns    eq : E, E → Bool
vars    x, y, z :→ E
eqns    eq(x, x) = true
        eq(x, y) = eq(y, x)
        eq(x, y) = true and eq(y, z) = true implies eq(x, z) = true

spec    NAT
uses    BOOL
sorts   N
opns    0 :→ N
        s : N → N
        equal : N, N → Bool
vars    n, m :→ N
eqns    equal(0, 0) = true
        equal(0, s(n) = false
        equal(s(n), 0) = false
        equal(s(n), s(m)) = equal(n, m)
```

Give a parametrized specification for sets over ELEMENT with the operations INSERT and REMOVE and prove:

1. The signature morphism σ : ELEMENT $\rightarrow$ NAT given by $\sigma(\mathrm{E}) = \mathrm{N}$ and $\sigma(\mathrm{eq} = \mathrm{equal})$ is no specification morphism.
2. $(T_{\mathrm{NAT}})|\sigma$ is a model of ELEMENT, i.e. it is a correct parameter assignment.
3. Does your specification satisfy $(T_{\mathrm{VALUE}})|_{\mathrm{NAT}} \cong T_{\mathrm{NAT}}$, i.e. is VALUE an extension of NAT? Is it an enrichment?

Exercise 28:

Make yourself familiar with the chapter on abstract reduction systems. Use the literature. Make sure you know proofs of lemmata 8.3, 8.5, theorem 8.6, theorem 8.16, lemma 8.18.

Exercise 29:

Consider the *mu*-calculus with the following rules for arbitrary $X, Y \in \{m, i, u\}^*$:

$$\{\frac{Xi}{Xiu}, \frac{mY}{mYY}, \frac{XiiiY}{XuY}, \frac{XuuY}{XY}\}$$

1. Is the reduction system it is based on terminating?
2. Do $mi \rightarrow mu$, $mu \rightarrow mi$ resp. hold? Prove your claim.

Have nice holidays!

Delivery: until 2008-01-07, better 2007-12-20, by EMail to Bernd Strieder