Exercises to the Lecture FSVT

Prof. Dr. Klaus Madlener   sheet 10

Exercise 30: [Confluence and termination of rule sets over ground terms]

Let $R = \{(l_k, r_k)|k = 1, \ldots, n)\}$ be a finite rule set over ground terms. Prove:

1. If there is an infinite chain, then there is a rule $(l, r) \in R$ with an infinite chain from r.

2. If there is an infinite chain, then there is a j with $1 \leq j \leq n$ and a ground term t, such that $l_j \overset{+}{\Rightarrow} t$ and l_j is a subterm of t.

3. Termination of R is decidable. (Termination is often denounced as 'Kettenbedingung' in german literature.)

4. Develop sufficient conditions for local confluence.

Exercise 31: [Knuth-Bendix-ordering]

Let $\varphi : F \cup V \to \mathbb{N}$ be a weight function with

$$
\begin{array}{lll}
\varphi(x) = \alpha > 0 & \text{for all } x \in V & (1) \\
\varphi(f) \geq \alpha & \text{if } f \text{ 0-ary} & (2) \\
\varphi(f) > 0 & \text{if } f \text{ 1-ary} & (3) \\
\varphi(f) \geq 0 & \text{else} & (4)
\end{array}
$$

Extend φ to $\varphi : \mathrm{Term}(F, V) \to \mathbb{N}$ by

$$\varphi(f(t_1, \ldots, t_n)) = \varphi(f) + \sum_{i=1,\ldots,n} \varphi(t_i)$$

Define $s > t$ iff. $\varphi(s) > \varphi(t)$ and $|s|_x \geq |t|_x$ for all $x \in V$. Then $>$ is called a Knuth-Bendix-ordering. Prove for any Knuth-Bendix-ordering $>$:

1. $>$ is strict part of a wellfounded partial ordering

2. $>$ is compatible with substitution

3. $>$ is compatible with term replacement

Exercise 32:

Let

$$R_1 = \{F(0, 1, x) \to F(x, x, x)\}$$
$$R_2 = \{G(x, y) \to x, G(x, y) \to y\}.$$

1. Prove: R_1 and R_2 are terminating.

2. Prove or disprove: The rule set $R_1 \cup R_2$ is terminating.

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