

## Exercises to the Lecture FSVT

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sheet 8

**Exercise 23:**Let  $\leq \subseteq \text{Term}(F, V) \times \text{Term}(F, V)$  be defined as: $s \lesssim t$  iff. exists a substitution  $\sigma$  with  $t \equiv \sigma(s)$  $s \approx t$  iff.  $s \lesssim t$  and  $t \lesssim s$  $s < t$  iff.  $s \lesssim t$  and  $s \not\approx t$ 

Show:

1.  $<$  is strict part of a well-founded partial order. Which elements is this partial order defined on?
2.  $s \approx t$  holds iff. a permutation  $\xi$  exists with  $s \equiv \xi(t)$  (variable renaming).

**Exercise 24:**This exercise is on an alternative specification of the integers  $\text{INTEGER} = (\text{sig}, E)$  with

$$\text{sig} = (\text{int}, 0, \text{succ}, \text{pred}, \text{add}),$$

$$E = \{\text{succ}(\text{pred}(x)) = x, \text{pred}(\text{succ}(x)) = x, \text{add}(0, y) = y, \text{add}(\text{succ}(x), y) = \text{succ}(\text{add}(x, y))\}$$

1. Show, that  $(\mathbb{Z}, 0, +1, -1, +)$  is initial in  $\text{Alg}(\text{INTEGER})$ .
2. Structurize this specification using the specification  $\text{INT}$ . Show that  $\text{INTEGER}$  is an enrichment of  $\text{INT}$ .
3. Extend  $\text{INTEGER}$  by a function `absolute` with the properties of the absolute value function on  $\mathbb{Z}$ . Show that this is an enrichment of  $\text{INT}$ .

**Exercise 25:**Let  $\text{sig}_1 = (\{\text{NAT}, \text{EVEN}\}, \{0, 1, S, f\}, \{0 : \rightarrow \text{NAT}, 1 : \rightarrow \text{EVEN}, S : \text{NAT} \rightarrow \text{NAT}, f : \text{NAT} \rightarrow \text{EVEN}\})$ . Further, let the  $\text{sig}_1$ -Algebra  $\mathfrak{A}_1$  be defined by:

$$A_{1, \text{NAT}} = \mathbb{N}, A_{1, \text{EVEN}} = 2\mathbb{N} \cup \{1\}, 0_{\mathfrak{A}_1} = 0, 1_{\mathfrak{A}_1} = 1, S_{\mathfrak{A}_1}(x) = x + 1, f_{\mathfrak{A}_1}(x) = \begin{cases} x, & \text{if } x \text{ even} \\ 1, & \text{else} \end{cases}$$

Prove:

1. There is no specification  $\text{spec}_1 = (\text{sig}_1, E_1)$  with finite  $E_1$ , such that  $T_{\text{spec}_1} \cong \mathfrak{A}_1$ .
2. There is a specification  $\text{spec}_2 = (\text{sig}_2, E_2)$  with  $\text{sig}_1 \subseteq \text{sig}_2$ ,  $E_2$  finite, such that  $T_{\text{spec}_2}|_{\text{sig}_1} \cong \mathfrak{A}_1$ .

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