Exercises to the Lecture FSVT

Prof. Dr. Klaus Madlener

Exercise 26:

Let INT2 be the specification of integers from example 7.9 of the lecture. We combine INT2 with BOOL and $((\{\}, \{<\}), E)$ to obtain a specification INT3, where

$E = \{<(0, \mathrm{succ}(x)) = \mathrm{true}, <(\mathrm{pred}(x), 0) = \mathrm{true}, <(0, \mathrm{pred}(x)) = \mathrm{false}, <(\mathrm{succ}(x), 0) = \mathrm{false}, <(\mathrm{pred}(x), \mathrm{pred}(y)) = <(x, y), <(\mathrm{succ}(x), \mathrm{succ}(y)) = <(x, y)\}$

1. Check, whether $T_{\mathrm{INT3}}\ |_{\mathrm{bool}} \cong \mathrm{Bool}$. Why would this be important? Hint: Look at $<(\mathrm{succ}(\mathrm{pred}(x)), \mathrm{pred}(\mathrm{succ}(y)))$.
2. Show that INT3 can not be fixed by additional equations.
3. Find further problems of INT3.
4. Make a suggestion for a specification INT4, such that $T_{\mathrm{INT4}}|_{\mathrm{int}} \cong \mathbb{Z}$, $T_{\mathrm{INT4}}|_{\mathrm{bool}} \cong$ Bool and $<$ is properly defined by its equations. Hint: Consider further function symbols.

Exercise 27:

Let specifications ELEMENT and NAT be given as:

`spec`	ELEMENT
`uses`	BOOL
`sorts`	E
`opns`	$\mathrm{eq} : \mathrm{E}, \mathrm{E} \to \mathrm{Bool}$
`vars`	$\mathrm{x}, \mathrm{y}, \mathrm{z} :\to \mathrm{E}$
`eqns`	$\mathrm{eq}(\mathrm{x}, \mathrm{x}) = \mathrm{true}$
	$\mathrm{eq}(\mathrm{x}, \mathrm{y}) = \mathrm{eq}(\mathrm{y}, \mathrm{x})$
	$\mathrm{eq}(\mathrm{x}, \mathrm{y}) = \mathrm{true}$ and $\mathrm{eq}(\mathrm{y}, \mathrm{z}) = \mathrm{true}$ implies $\mathrm{eq}(\mathrm{x}, \mathrm{z}) = \mathrm{true}$

`spec`	NAT
`uses`	BOOL
`sorts`	N
`opns`	$0 :\to \mathrm{N}$
	$\mathrm{s} : \mathrm{N} \to \mathrm{N}$
	$\mathrm{equal} : \mathrm{N}, \mathrm{N} \to \mathrm{Bool}$
`vars`	$\mathrm{n}, \mathrm{m} :\to \mathrm{N}$
`eqns`	$\mathrm{equal}(0, 0) = \mathrm{true}$
	$\mathrm{equal}(0, \mathrm{s}(\mathrm{n}) = \mathrm{false}$
	$\mathrm{equal}(\mathrm{s}(\mathrm{n}), 0) = \mathrm{false}$
	$\mathrm{equal}(\mathrm{s}(\mathrm{n}), \mathrm{s}(\mathrm{m})) = \mathrm{equal}(\mathrm{n}, \mathrm{m})$

Give a parametrized specification for sets over ELEMENT with the operations INSERT and REMOVE and prove:

1. The signature morphism σ : ELEMENT $\rightarrow$ NAT given by $\sigma(\mathrm{E}) = \mathrm{N}$ and $\sigma(\mathrm{eq} =$ equal) is no specification morphism.
2. $(T_{\mathrm{NAT}})|\sigma$ is a model of ELEMENT, i.e. it is a correct parameter assignment.
3. Does your specification satisfy $(T_{\mathrm{VALUE}})|_{\mathrm{NAT}} \cong T_{\mathrm{NAT}}$, i.e. is VALUE an extension of NAT? Is it an enrichment?

Exercise 28:

Make yourself familiar with the chapter on abstract reduction systems. Use the literature. Make sure you know proofs of lemmata 8.3, 8.5, theorem 8.6, theorem 8.16, lemma 8.18.

Exercise 29:

Consider the *mu*-calculus with the following rules for arbitrary $X, Y \in \{m, i, u\}^*$:

$$\{\frac{Xi}{Xiu}, \frac{mY}{mYY}, \frac{XiiiY}{XuY}, \frac{XuuY}{XY}\}$$

1. Is the reduction system it is based on terminating?
2. Do $mi \rightarrow mu$, $mu \rightarrow mi$ resp. hold? Prove your claim.

Have nice holidays!

Delivery: until 2008-12-18, Fr:G11 Mo:G02,
by EMail to madlener@informatik.uni-kl.de