

Formal Specification and Verification Techniques

Prof. Dr. K. Madlener

21. Oktober 2010

Course of Studies „Informatics“, „Applied Informatics“ and
 „Master-Inf.“ WS10/11
 Prof. Dr. Madlener
 TU- Kaiserslautern

Lecture:

Mo 08.15–09.45 48-462

We 08.15–09.45 48-462

Exercises:??

Fr. 11.45–13.15 32-439

Mo 13.45–15.45 32-439

- ▶ Information <http://www-madlener.informatik.uni-kl.de/teaching/ss2009/fsvt/fsvt.html>
- ▶ Evaluation method:
Exercises (efficiency statement) + Final Exam (Credits)
- ▶ First final exam: (Written or Oral)
- ▶ Exercises (Dates and Registration): See WWW-Site

Bibliography



M. O'Donnell.

Computing in Systems described by Equations, LNCS 58, 1977.
Equational Logic as a Programming language.



J. Avenhaus.

Reduktionssysteme, (Skript), Springer 1995.



Cohen et.al.

The Specification of Complex Systems.



Bergstra et.al.

Algebraic Specification.



Barendregt.

Functional Programming and Lambda Calculus. Handbook of TCS,
321-363, 1990.

Bibliography



Gehani et.al.

Software Specification Techniques.



Huet.

Confluent Reductions: Abstract Properties and Applications to TRS,
JACM, 27, 1980.



Nivat, Reynolds.

Algebraic Methods in Semantics.



Loeckx, Ehrich, Wolf.






Specification of Abstract Data Types, Wyley-Teubner, 1996.



J.W. Klop.

Term Rewriting System. Handbook of Logic, INCS, Vol. 2, Abransky,
Gabbay, Maibaum.

Bibliography

-  Ehrig, Mahr.
Fundamentals of Algebraic Specification.
-  Peyton-Jones.
The Implementation of Functional Programming Language.
-  Plasmeister, Eekelen.
Functional Programming and Parallel Graph Rewriting.
-  Astesiano, Kreowski, Krieg-Brückner.
Algebraic Foundations of Systems Specification (IFIP).
-  N. Nissanke.
Formal Specification Techniques and Applications (Z, VDM, algebraic), Springer 1999.

Bibliography



Turner, McCluskey.

The construction of formal specifications. (Model based (VDM) + Algebraic (OBJ)).



Goguen, Malcom.

Algebraic Semantics of Imperative Programs.



H. Dörr.





Efficient Graph Rewriting and its Implementation.



B. Potter, J. Sinclair, D. Till.

An introduction to Formal Specification and Z. Prentice Hall, 1996.

Bibliography

-  [J. Woodcok, J. Davis.](#)
Using Z: Specification, Refinement and Proof, Prentice Hall 1996.
-  [J.R. Abrial.](#)
The B-Book; Assigning Programs to Meanings. Cambridge U. Press, 1996.
-  [E. Börger, R. Stärk](#)
Abstract State Machines: A Method for High-Level System Design and Analysis. Springer, 2003.
-  [F. Baader, T. Nipkow](#)
Term Rewriting and All That. Cambridge, 1999.

Goals - Contents

General Goals:

Formal foundations of Methods
for Specification, Verification and Implementation

Summary

- ▶ The Role of formal Specifications
- ▶ Abstract State Machines: ASM-Specification methods
- ▶ Algebraic Specification, Equational Systems
- ▶ Reduction systems, Term Rewriting Systems
- ▶ Equational - Calculus and - Programming
- ▶ Related Calculi: λ -Calculus, Combinator- Calculus
- ▶ Implementation, Reduction Strategies, Graph Rewriting

Lecture's Contents

Role of formal Specifications

Motivation

Properties of Specifications

Formal Specifications

Examples

Abstract State Machines (ASMs)

Abstract State Machines: ASM- Specification's method

- Fundamentals

- Sequential algorithms

- ASM-Specifications

Distributed ASM: Concurrency, reactivity, time

- Fundamentals: Orders, CPO's, proof techniques

- Induction

- DASM

- Reactive and time-depending systems

Refinement

- Lecture Börger's ASM-Buch

Algebraic Specification

Algebraic Specification - Equational Calculus

Fundamentals

Introduction

Algebrae

Algebraic Fundamentals

Signature - Terms

Strictness - Positions- Subterms

Interpretations: sig-algebras

Canonical homomorphisms

Equational specifications

Substitution

Loose semantics

Connection between $\models, =_E, \vdash_E$

Birkhoff's Theorem

Algebraic Specification: Initial Semantics

Initial semantics

- Basic properties

- Correctness and implementation

- Structuring mechanisms

- Signature morphisms - Parameter passing

- Semantics parameter passing

- Specification morphisms

Algebraic Specification: operationalization

Reduction Systems

- Abstract Reduction Systems

- Principle of the Noetherian Induction

- Important relations

- Sufficient conditions for confluence

- Equivalence relations and reduction relations

- Transformation with the inference system

- Construction of the proof ordering

Term Rewriting Systems

- Principles

- Critical pairs, unification

- Local confluence

- Confluence without Termination

- Knuth-Bendix Completion

Computability and Implementation

Equational calculus and Computability

Implementations

Primitive Recursive Functions

Recursive and partially recursive functions

Partial recursive functions and register machines

Computable algebrae

Reduction strategies

Generalities

Orthogonal systems

Strategies and length of derivations

Sequential Orthogonal TES: Call by Need

Summary

Summary

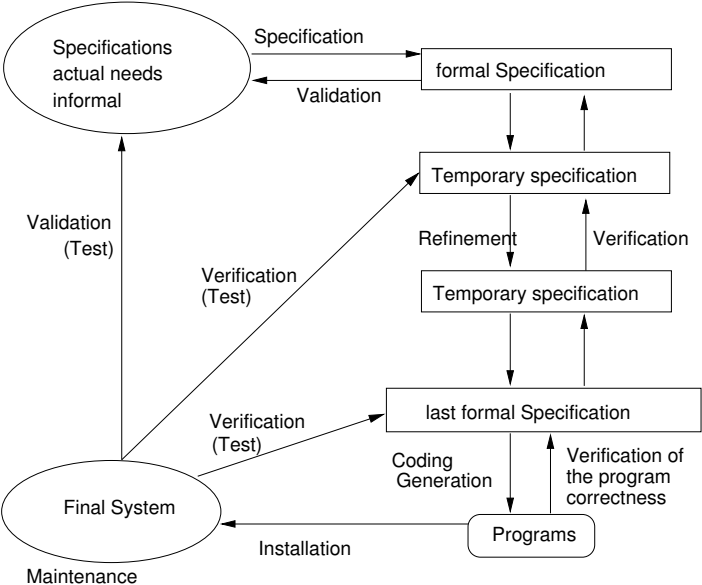
Models for SW-Development

- ▶ Waterfall model, Spiral model, . . .

Phases \equiv Activities + Product Parts (partial descriptions)

In each stage of the DP

Description: a SW specification, that is, a stipulation of what must be achieved, but not always how it is done.



Properties of Specifications

Consistency

Completeness

- ▶ **Validation** of the global specification regarding the requirements.
- ▶ **Verification** of intermediate specifications regarding the previous one.
- ▶ **Verification** of the programs regarding the specification.
- ▶ **Verification** of the integrated final system with respect to the global specification.
- ▶ **Activities:** Validation, Verification, Testing
Consistency- and Completeness-Check
- ▶ **Tool support** needed!

Requirements

Functional -
what
:
how

- **non functional**
time aspects
robustness
stability
security
adaptability
ergonomics
maintainability

Properties

Correctness: Does the implemented System fulfill the Requirements?

Test

Validate

Verify

Validation - Verification

From Wikipedia, the free encyclopedia

In common usage, **validation** is the process of checking if something satisfies a certain criterion. Examples would include checking if a statement is true (validity), if an appliance works as intended, if a computer system is secure, or if computer data are compliant with an open standard. Validation implies one is able to document that a solution or process is correct or is suited for its intended use.

In engineering or as part of a quality management system, **validation** confirms that the needs of an external customer or user of a product, service, or system are met. **Verification** is usually an internal quality process of determining compliance with a regulation, standard, or specification. An easy way of recalling the difference between validation and verification is that

validation is ensuring “you built the right product” and

verification is ensuring “you built the product right.”

Validation is testing to confirm that it satisfies user’s needs.

Requirements

- ▶ The **global specification** describes, as exact as possible, what must be done.

- ▶ **Abstraction of the *how***

Advantages

- ▶ *apriori*: Reference document, compact and legible.
 - ▶ *aposteriori*: Possibility to follow and document design decisions \rightsquigarrow
traceability, reusability, maintenance.
- ▶ **Problem**: Size and complexity of the systems.

Principles to be supported

- ▶ **Refinement principle**: Abstraction levels
- ▶ **Structuring mechanisms**
Decomposition and modularization principles
- ▶ Object orientation
- ▶ **Verification and validation concepts**

Requirements Description \rightsquigarrow Specification Language

- ▶ Choice of the specification technique depends on the System.
Frequently more than a single specification technique is needed.
(What – How).
- ▶ Type of Systems:
Pure function oriented (I/O), reactive- embedded- real time-
systems.
- ▶ **Problem** : Universal Specification Technique (UST)
difficult to understand, ambiguities, tools, size ...
e.g. UML
- ▶ **Desired**: Compact, legible and exact specifications

Here: **formal specification techniques**

Formal Specifications

- ▶ A specification in a formal specification language defines all the possible behaviors of the specified system.
- ▶ 3 Aspects: **Syntax, Semantics, Inference System**
 - ▶ **Syntax**: What's allowed to write: Text with structure, Properties often described by formulas from a logic.
 - ▶ **Semantics**: Which models are associated with the specification, \rightsquigarrow specification models.
 - ▶ **Inference System**: Consequences (Derivation) of properties of the system. \rightsquigarrow Notion of consequence.

Formal Specifications

- ▶ Two main classes:

Model oriented

(constructive)

e.g. VDM, Z, ASM

Construction of a
non-ambiguous model

from available

data structures and

construction rules

Concept of correctness

- -

Property oriented

(declarative)

signature (functions, predicates)

Properties

(formulas, axioms)

models

algebraic specification

AFFIRM, OBJ, ASF, ...

- ▶ Operational specifications:
Petri nets, process algebras, automata based (SDL).

Specifications: What for?

- ▶ The concept of program correctness is not well defined without a formal specification.
- ▶ A verification is not possible without a formal specification.
- ▶ Other concepts, like the concept of refinement, simulation become well defined.

Wish List

- ▶ Small gap between specification and program:
[Generators](#), [Transformators](#).
- ▶ Not too many different formalisms/notations.
- ▶ Tool support.
- ▶ Rapid prototyping.
- ▶ Rules for “constructing” specifications, that guarantee certain properties (e.g. consistency + completeness).

Formal Specifications

- ▶ Advantages:
 - ▶ The concepts of correctness, equivalence, completeness, consistency, refinement, composition, etc. are treated in a mathematical way (based on the logic)
 - ▶ Tool support is possible and often available
 - ▶ The application and interconnection of different tools are possible.
- ▶ Disadvantages:

Refinements

Abstraction mechanisms

- ▶ Data abstraction (representation)
- ▶ Control abstraction (Sequence)
- ▶ Procedural abstraction (only I/O description)

Refinement mechanisms

- ▶ Choose a data representation (sets by lists)
- ▶ Choose a sequence of computation steps
- ▶ Develop algorithm (Sorting algorithm)

Concept: **Correctness of the implementation**

- ▶ Observable equivalences
- ▶ Behavioral equivalences

Structuring

Problems: Structuring mechanisms

► Horizontal:

Decomposition/Aggregation/Combination/Extension/
Parameterization/Instantiation
(Components)

Goal: Reduction of complexity, Completeness

► Vertical:

Realization of Behavior
Information Hiding/Refinement

Goal: Efficiency and Correctness

Tool support

- ▶ Syntactic support (grammars, parser,...)
- ▶ Verification: theorem proving (proof obligations)
- ▶ Prototyping (executable specifications)
- ▶ Code generation (out of the specifications generate C code)
- ▶ Testing (from the specification generate test cases for the program)

Desired:

To generate the tools out of the syntax and semantics of the specification language

Example: declarative

Example 2.1. *Restricted logic: e.g. equational logic*

- ▶ *Axioms:* $\forall X \ t_1 = t_2 \quad t_1, t_2 \text{ terms.}$
- ▶ *Rules:* *Equals are replaced with equals. (directed).*
- ▶ *Terms* \approx *names for objects (identifier), structuring, construction of the object.*
- ▶ *Abstraction:* *Terms as elements of an algebra, term algebra.*

Example: declarative

Foundations for the algebraic specification method:

- ▶ Axioms induce a **congruence** on a term algebra
- ▶ Independent subtasks
 - ▶ Description of properties with equality axioms
 - ▶ Representation of the terms
- ▶ Operationalization
 - ▶ spec, **t term** give out the „value“ of t , i.e. **$t' \in \text{Value}(\text{spec})$** with $\text{spec} \models t = t'$.
 - ▶ \rightsquigarrow **Functional programming**: LISP, CAML, ...
 $F(t_1, \dots, t_n) \quad \text{eval}() \rightsquigarrow \text{value}.$

Example: Model-based constructive: VDM

Unambiguous (Unique model), standard (notations),
Independent of the implementation, formally manipulable, abstract,
structured, expressive, consistency by construction

Example 2.2. *Model (state)-based specification technique VDM*

- ▶ Based on naive set theory, PL 1, preconditions and postconditions.

Primitive types: \mathbb{B} Boolean $\{true, false\}$
 \mathbb{N} natural $\{0, 1, 2, 3, \dots\}$, \mathbb{Z}, \mathbb{R}

- ▶ *Sets:* \mathbb{B} -Set: Sets of \mathbb{B} -'s.
- ▶ *Operations on sets:* \in : Element, Element-Set $\rightarrow \mathbb{B}$, \cup, \cap, \setminus
- ▶ *Sequences:* \mathbb{Z}^* : Sequences of integer numbers.
- ▶ *Sequence operations:* \frown : Sequences, Sequences \rightarrow Sequences.
„Concatenation“

e.g. $[] \frown [true, false, true] = [true, false, true]$

len: sequences $\rightarrow \mathbb{N}$, *hd:* sequences \rightsquigarrow elem (partial).

tl: sequences \rightsquigarrow sequences, *elem:* sequences \rightarrow Elem-Set.

Operations in VDM

See e.g.: <http://www.vdmportal.org/twiki/bin/view/VDM-SL: System State, Specification of operations>

Format:

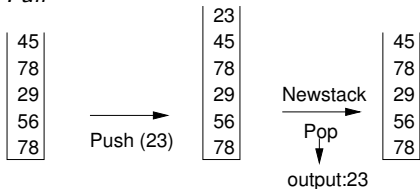
Operation-Identifier (Input parameters) Output parameters
 Pre-Condition
 Post-Condition

e.g.

$Int_SQR(x : \mathbb{N})z : \mathbb{N}$
 pre $x \geq 1$
 post $(z^2 \leq x) \wedge (x < (z + 1)^2)$

Example VDM: Bounded stack

Example 2.3. ▶ *Operations:* · *Init* · *Push* · *Pop* · *Empty* ·
Full



Contents = \mathbb{N}^* Max_Stack_Size = \mathbb{N}

▶ STATE STACK OF

s : Contents

n : Max_Stack_Size

inv : mk-STACK(s, n) \triangleq len $s \leq n$

END

Bounded stack

```

Init(size : ℕ)
ext wr s : Contents
  wr n : Max_Stack_Size
pre true
post s = [ ] ∧ n = size

```

```

Push(c : ℕ)
ext wr s : Contents
  rd n : Max_Stack_Size
pre len s < n
post s = [c] ∪  $\overleftarrow{s}$ 

```

```

Full() b : ℬ
ext rd s : Contents
  rd n : Max_Stack_Size
pre true
post b ⇔ (len s = n)

```

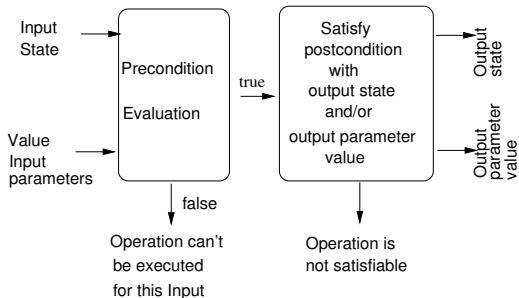
```

Pop() c : ℕ
ext wr s : Contents
pre len s > 0
post  $\overleftarrow{s}$  = [c] ∩ s

```

↪ **Proof-Obligations**

General format for VDM-operations



General form VDM-operations

Proof obligations:

For each acceptable input there's (at least) one acceptable output.

$$\forall s_i, i \cdot (\text{pre-op}(i, s_i) \Rightarrow \exists s_o, o \cdot \text{post-op}(i, s_i, o, s_o))$$

When there are state-invariants at hand:

$$\forall s_i, i \cdot (\text{inv}(s_i) \wedge \text{pre-op}(i, s_i) \Rightarrow \exists s_o, o \cdot (\text{inv}(s_o) \wedge \text{post-op}(i, s_i, o, s_o)))$$

alternatively

$$\forall s_i, i, s_o, o \cdot (\text{inv}(s_i) \wedge \text{pre-op}(i, s_i) \wedge \text{post-op}(i, s_i, o, s_o) \Rightarrow \text{inv}(s_o))$$

See e.g. Turner, McCluskey The Construction of Formal Specifications
or Jones C.B. Systematic SW Development using VDM Prentice Hall.

Stack: algebraic specification

Example 2.4. Elements of an algebraic specification: *Signature* (sorts, operation names with the arity), *Axioms* (often only equations)

SPEC STACK

USING NATURAL, BOOLEAN “Names of known SPECS”

SORT stack “Principal type”

OPS *init* : \rightarrow stack “Constant of the type stack, empty stack”

push : stack nat \rightarrow stack

pop : stack \rightarrow stack

top : stack \rightarrow nat

is_empty? : stack \rightarrow bool

stack_error : \rightarrow stack

nat_error : \rightarrow nat

(*Signature* fixed)

Axioms for Stack

FORALL $s : \text{stack} \quad n : \text{nat}$

AXIOMS

$\text{is_empty? (init)} = \text{true}$
 $\text{is_empty? (push (s, n))} = \text{false}$
 $\text{pop (init)} = \text{stack_error}$
 $\text{pop (push (s, n))} = s$
 $\text{top (init)} = \text{nat_error}$
 $\text{top (push (s,n))} = n$

Terms or expressions:

$\text{top (push (push (init, 2), 3))}$ “means” 3

How is the “bounded stack” specified algebraically?

Semantics? Operationalization?

Variant: Z and B- Methods: Specification-Development-Programs.

- ▶ **Covering:** Technical specification (what), development through refinement, architecture (layers' architecture), generation of executable code.
- ▶ **Proofs:** Program construction \equiv Proof construction.
Abstraction, instantiation, decomposition.
- ▶ **Abstract machines:** Encapsulation of information (Modules, Classes, ADT).
- ▶ **Data and operations:** SWS is composed of abstract machines.
Abstract machines „get “ data and „offer“ operations.
Data can only be accessed through operations.

Z- and B- Methods: Specification-Development-Programs.

- ▶ **Data specification:** Sets, relations, functions, sequences, trees. Rules (static) with help of invariants.
- ▶ **Operator specification:** not executable „pseudocode“.
 - Without loops:
 - Precondition + atomic action
 - PL1 generalized substitution
- ▶ **Refinement** (\rightsquigarrow implementation).
- ▶ **Refinement** (as specification technique).
- ▶ **Refinement techniques:**
 - Elimination of not executable parts, introduction of control structures (cycles).
 - Transformation of abstract mathematical structures.

Z- and B- Methods: Specification-Development-Programs.

- ▶ **Refinement steps:** Refinement is done in several steps.
Abstract machines are newly constructed. Operations for users remain the same, only internal changes.
In-between steps: Mix code.
- ▶ **Nested architecture:**
Rule: not too many refinement steps, better apply decomposition.
- ▶ **Library:** Predefined abstract machines, encapsulation of classical DS.
- ▶ **Reusability**
- ▶ **Code generation:** Last abstract machine can be easily translated into a program in an imperative Language.

Z- and B- Methods: Specification-Development-Programs.

Important here:

- ▶ **Notation:** Theory of sets + PL1, standard set operations, Cartesian product, power sets, set restrictions $\{x \mid x \in s \wedge P\}$, P predicate.
- ▶ **Schemata (Schemes)** in Z Models for declaration and constraint {state descriptions}.
- ▶ **Types.**
- ▶ **Natural Language:** Connection Math objects \rightarrow objects of the modeled world.
- ▶ See Abrial: The B-Book,
Potter, Sinclair, Till: An Introduction to Formal Specification and Z,
Woodcock, Davis: Using Z Specification, Refinement, and Proof \rightsquigarrow

Literature