SS 2010 22.11.2010

Exercises to the Lecture FSVT

Prof. Dr. Klaus Madlener sheet 5

**Exercise 1:**

1. Prove or disprove correctness of the abstract version of termination detection algorithm from slide 156.
2. Answer the question from slide 158, whether the given DASM on the termination detection problem is a refinement of the more abstract DASM. Take the problems resolved into consideration.

**Exercise 2:**

1. Give models for the specifications `NAT` and `LIST(NAT)` from the lecture, where the sets of support consist of ground terms.
2. Give models for the specifications `NAT` and `LIST(NAT)`, with + not commutative, and app not associative.

   Are your sig-algebras term-generated?

**Exercise 3:**

Let the specification `LIST(NAT)` $= (\mathrm{sig}, E)$ be the specification of lists from the lecture.

1. Show, that for every ground term, there is a $E$-equal ground term not containing `app`.
2. Show: $\mathrm{app}(q_1, \mathrm{app}(q_2, q_3)) = \mathrm{app}(\mathrm{app}(q_1, q_2), q_3) \in \mathrm{ITH}(E)$

**Exercise 4:**

Prove:

1. Let $t, t', t'' \in \mathrm{Term}(F, V), u \in O(t), v \in O(t')$. Then holds:

$$t[u \leftarrow t']/uv \equiv t'/v \qquad \text{(embedding)}$$
$$t[u \leftarrow t'][uv \leftarrow t''] \equiv t[u \leftarrow t'[v \leftarrow t'']] \qquad \text{(associativity)}$$

   or in alternative syntax:

$$t[t']_u \mid_{uv} \equiv t' \mid_v \qquad \text{(embedding)}$$
$$t[t']_u[t'']_{uv} \equiv t[t'[t'']_v]_u \qquad \text{(associativity)}$$

2. Let $t, t', t'' \in \text{Term}(F, V), u, v \in O(t), u \mid v$ ($u, v$ are disjunct positions, i.e. neither $u$ is prefix of $v$ nor $v$ prefix of $u$). Then holds:

$$t[u \leftarrow t']/v \equiv t/v \qquad \text{(persistence)}$$
$$t[u \leftarrow t'][v \leftarrow t''] \equiv t[v \leftarrow t''][u \leftarrow t'] \qquad \text{(commutativity)}$$

3. Let $t, t', t'' \in \text{Term}(F, V), u, v, w \in O(t), u = vw$. Then holds:

$$t[u \leftarrow t']/v \equiv (t/v)[w \leftarrow t'] \qquad \text{distributivity}$$
$$t[u \leftarrow t'][v \leftarrow t''] \equiv t[v \leftarrow t''] \qquad \text{(dominance)}$$

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