SS 2010 07.12.2010

Exercises to the Lecture FSVT

Prof. Dr. Klaus Madlener sheet 7

Exercise 1:

Let specifications ELEMENT and NAT be given as:

`spec`	ELEMENT
`uses`	BOOL
`sorts`	E
`opns`	$\mathrm{eq} : \mathrm{E}, \mathrm{E} \to \mathrm{Bool}$
`vars`	$\mathrm{x}, \mathrm{y}, \mathrm{z} :\to \mathrm{E}$
`eqns`	$\mathrm{eq}(\mathrm{x}, \mathrm{x}) = \mathrm{true}$
	$\mathrm{eq}(\mathrm{x}, \mathrm{y}) = \mathrm{eq}(\mathrm{y}, \mathrm{x})$
	$\mathrm{eq}(\mathrm{x}, \mathrm{y}) = \mathrm{true}$ and $\mathrm{eq}(\mathrm{y}, \mathrm{z}) = \mathrm{true}$ implies $\mathrm{eq}(\mathrm{x}, \mathrm{z}) = \mathrm{true}$

`spec`	NAT
`uses`	BOOL
`sorts`	N
`opns`	$0 :\to \mathrm{N}$
	$\mathrm{s} : \mathrm{N} \to \mathrm{N}$
	$\mathrm{equal} : \mathrm{N}, \mathrm{N} \to \mathrm{Bool}$
`vars`	$\mathrm{n}, \mathrm{m} :\to \mathrm{N}$
`eqns`	$\mathrm{equal}(0, 0) = \mathrm{true}$
	$\mathrm{equal}(0, \mathrm{s}(\mathrm{n}) = \mathrm{false}$
	$\mathrm{equal}(\mathrm{s}(\mathrm{n}), 0) = \mathrm{false}$
	$\mathrm{equal}(\mathrm{s}(\mathrm{n}), \mathrm{s}(\mathrm{m})) = \mathrm{equal}(\mathrm{n}, \mathrm{m})$

Give a parametrized specification for sets over ELEMENT with the operations INSERT and REMOVE and prove:

1. The signature morphism σ : ELEMENT $\to$ NAT given by $\sigma(\mathrm{E}) = \mathrm{N}$ and $\sigma(\mathrm{eq} = \mathrm{equal})$ is no specification morphism.
2. $(T_{\mathrm{NAT}})|\sigma$ is a model of ELEMENT, i.e. it is a correct parameter assignment.
3. Does your specification satisfy $(T_{\mathrm{VALUE}})|_{\mathrm{NAT}} \cong T_{\mathrm{NAT}}$, i.e. is VALUE an extension of NAT? Is it an enrichment?

Exercise 2:

Consider the *mu*-calculus with the following rules for arbitrary $X, Y \in \{m, i, u\}^*$:

$$\{\frac{Xi}{Xiu}, \frac{mY}{mYY}, \frac{XiiiY}{XuY}, \frac{XuuY}{XY}\}$$

1. Is the reduction system it is based on terminating?
2. Do $mi \to mu$, $mu \to mi$ resp. hold? Prove your claim.

Exercise 3:

Prove the properties of the muiltiset ordering following definition 8.22 on slide 284.

Exercise 4:

Prove that the proof ordering from slide 285 is noetherian.

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