

## Exercises to the Lecture FSVT

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sheet 10

**Exercise 1:** [Implementation by equations]

Let  $g : \mathbb{N}^{n+1} \rightarrow \mathbb{N}$ ,  $h : \mathbb{N}^n \rightarrow \mathbb{N}$  be primitively recursive functions and let  $f : \mathbb{N}^n \rightarrow \mathbb{N}$  be defined by:

$$f(x_1, \dots, x_n) = \mu_{z \leq h(x_1, \dots, x_n)} [g(x_1, \dots, x_n, z) = 0]$$

Give an equation set  $G_{\hat{f}}$  and a function symbol  $\hat{f}$ , such that  $\hat{f}$  implements the function  $f$  in  $G_{\hat{f}}$ .

**Exercise 2:** [PL-Semantik]

Let the following grammar generating PL-programs over the symbols  $\Sigma = \{A, B, \dots, Z, 0, 1, \dots, 9, \leftarrow, +, :, ;, \mathbf{loop}, \mathbf{end}, \mathbf{goto}\}$  be given:

$$\begin{aligned} \langle \text{letter} \rangle &\rightarrow A \mid B \mid \dots \mid Z \\ \langle \text{digit} \rangle &\rightarrow 0 \mid 1 \mid \dots \mid 9 \\ \langle \text{name} \rangle &\rightarrow \langle \text{letter} \rangle \mid \langle \text{name} \rangle \langle \text{letter} \rangle \mid \langle \text{name} \rangle \langle \text{digit} \rangle \\ \langle \text{assignment} \rangle &\rightarrow \langle \text{name} \rangle \leftarrow 0 \mid \langle \text{name} \rangle \leftarrow \langle \text{name} \rangle + 1 \mid \langle \text{name} \rangle \leftarrow \langle \text{name} \rangle \\ \langle \text{command} \rangle &\rightarrow \langle \text{assignment} \rangle \mid \mathbf{goto} \langle \text{name} \rangle \\ \langle \text{mark. command} \rangle &\rightarrow \langle \text{command} \rangle ; \mid \langle \text{name} \rangle : \langle \text{command} \rangle \\ \langle \text{loop beginning} \rangle &\rightarrow \mathbf{loop} \langle \text{name} \rangle ; \mid \langle \text{name} \rangle : \mathbf{loop} \langle \text{name} \rangle ; \\ \langle \text{loop end} \rangle &\rightarrow \mathbf{end} ; \mid \langle \text{name} \rangle : \mathbf{end} ; \\ \langle \text{program} \rangle &\rightarrow \langle \text{mark. command} \rangle \\ &\quad \mid \langle \text{loop beginning} \rangle \langle \text{program} \rangle \langle \text{loop end} \rangle \\ &\quad \mid \langle \text{program} \rangle \langle \text{program} \rangle \end{aligned}$$

1. Give an operational semantic of PL-programs by a function  $M_e : P \times \mathbb{N}^k \rightarrow \mathbb{N}^k$ , where  $k$  is the number of variables occurring in the program.
2. Determine suitable term sets, an equation set  $G$  and a function EVAL implementing  $M_e$  in  $G$ .

Hints: To make it simpler, sensible restrictions or extensions of the grammar above may be assumed, e.g. every command is marked, the marks are natural numbers.

One possible approach is representing commands as constants (e.g.  $c_{i0} \equiv x_i \leftarrow 0$ ,  $c_{ij} \equiv x_i \leftarrow x_j$ ,  $c'_{ij} \equiv x_i \leftarrow x_j + 1$ ) and programs as lists of commands.

Examine special cases like jumping out of and into loops.

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